Imperfect Credit Markets, Household Wealth Distribution, and Development

By

Kiminori Matsuyama

Department of Economics, Northwestern University

Homepage: http://www.faculty.econ.northwestern.edu/faculty/matsuyama/

Email: k-matsuyama@northwestern.edu

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ABSTRACT: This article discusses some key results in the theoretical literature on credit market imperfections, household wealth distribution and development by conducting three types of analysis, which progressively build on one another. The first, a single dynasty model, explains how a household may be caught in a poverty trap due to credit market imperfections, but can say little about the effects of distribution on development. The second, a model of interacting dynasties with a fixed threshold, explains a collective poverty trap, with path-dependence in the wealth distribution dynamics, but can say little about the effects of inequality on development, due to its absolute notion of the rich and the poor. The third, models of interacting dynasties with variable thresholds, offers a richer framework for understanding the dynamics of inequality and development under credit market imperfections, due to its relative notion of the rich and poor.

KEYWORDS: Inequality and Growth, Individual versus Collective Poverty Traps, Path-dependence, Trickle-Down, Symmetry-Breaking, Emergent versus Dissipating Class Structures

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1. Introduction

Much has been written on the question of how credit constraints affect the intertwining nature of household wealth distribution and development dynamics. Yet, there seem to be significant confusion and numerous misconceptions in the literature. This article aims to highlight some key results in the theoretical literature by using a series of illustrative models in an attempt to dispel some commonly held misconceptions. To this end, it is useful to distinguish three types of analysis.[1]

The first type is what I call "single dynasty models", of which Galor & Zeira (1993) is a notable example. It looks at the wealth dynamics of a single household in isolation, and shows the possibility of an "individual poverty trap." Due to credit constraints, the household needs to be wealthy enough to be able to finance the profitable investment (such as setting up a company or obtaining higher education). This implies that a poor household, who starts below the threshold, may be caught in vicious circles of poverty, or at least will have difficulty getting out of poverty. This alone suggests that wealth inequality might rise at least initially over the course of development, generating something akin to the Kuznets Inverted U-curve. Contrary to the claim often found in the literature, this type of analysis cannot make any clear prediction regarding the effects of wealth distribution in general, and wealth inequality in particular, on the development process. This is because household wealth dynamics are independent of household wealth distribution in the economy.

The second type is what I call "models of interacting dynasties with fixed thresholds", of which Banerjee & Newman (1993) is a notable example. A rich household, whose wealth is above the fixed threshold, finances to set up a firm and become an employer. A poor household,

[1] I apologize in advance to those researchers whose research could not be cited due to the space limitations. Nor does this article do justice to all the richness and subtleties of the research cited for the sake of keeping the discussion focused. See the reference listed under Related Resources for surveys with different focuses or on some closely related research.

whose wealth is below the fixed threshold, cannot finance a firm, and has no choice but to work for the rich. If too many households are below the threshold, this depresses the wage rate, which makes it harder for the poor workers to get out of poverty. On the other hand, if sufficiently many households start above the threshold, their labor demand drives up the wage rate, which pulls the poor workers out of poverty. This eventually leads to a general prosperity. In other words, there is a "collective poverty trap"; the poor remains poor, not so much because they are poor but because there are too many of them. In this type of analysis, the dynamics of wealth distribution exhibits path-dependence. Although this is often interpreted as "persistence of inequality," greater initial inequality does not necessarily imply greater long run inequality. In fact, the opposite might be true; making initial distribution more unequal could help more households cross the threshold, thereby ensuring long run equality. This type of analysis cannot make any clear prediction about the effects of wealth inequality, because its notion of the rich and the poor is an absolute one; what separates the two is a certain exogenous amount of wealth. The dynamics depends on a particular feature of the distribution, the share of the households above the fixed threshold, which may or may not increase with wealth inequality. This type of analysis also suggests the possibility of an "underdevelopment trap," where every household remains poor. If all households are below the threshold, nobody becomes an employer, which means that the wage rate becomes so low, which means that no household could ever cross the threshold. The presence of such equality in poverty, though often emphasized as an important message, is not robust as it critically depends on the assumption that the threshold is exogenously fixed.

This brings us to the third type, "models of interacting dynasties with variable thresholds," examples of which include Aghion & Bolton (1997), Freeman (1996), Matsuyama (2000, 2006), and Moohkerjee & Ray (2003, 2010). For example, in Matsuyama (2006), the threshold moves in response to wealth distribution because a potential borrower/employer could pledge a (possibly small) fraction of the profit to the lender; a lower wage means a higher profit, would raise the borrowing limit, and hence lower the threshold. With the varying threshold, the notion of the rich and the poor is relative in this model. What separates the two is their relative positions within the distributions. This eliminates the possibility of an underdevelopment trap. Furthermore, the model unveils two additional cases that are missing in models with the fixed threshold. The first is "symmetry-breaking," which occurs under the condition of relatively severe credit market imperfections and/or relatively low productivity. Starting from any initial distribution (i.e., even if it is perfectly equal), there will be long run inequality, where the population is split between the rich entrepreneurial class (the bourgeoisie) and the poor working class (the proletariat). This case suggests that a one-shot redistribution would be ineffective in eliminating inequality. The second is "global convergence," which occurs under the condition of relatively mild credit market imperfections and/or relatively high productivity. Starting from any initial distribution (i.e., even if it is highly unequal), there will be long run equality both in earning and in wealth between workers and employers. This case suggests that redistribution is unnecessary to achieve long run equality.

Perhaps the following thought experiment helps to clarify the distinction between the three types of analysis. Imagine that a large number of penniless families immigrate to this economy. What are the effects on those currently living in this economy, who are below the threshold (hence they are poor but not as poor as the immigrants)? The first type of analysis says "zero effect," because wealth distribution plays no role in household wealth dynamics. The second says "a negative effect," because there will be more households below the fixed threshold. The third says that it could have "a positive effect," because the arrival of penniless immigrants lowers the threshold.

2. A Single Dynasty Model; An Individual Poverty Trap

Let us start with a model of a single dynasty, which takes its environment as given. This model will also be used as a building block in the models of interacting dynasties to follow.

Time is discrete and extends from zero to infinity ($t = 0, 1, 2, \ldots$). There is a single numeraire good, which can be either consumed or invested. A dynasty (or a household) is an infinite-sequence of one-period lived agents linked by inheritance. At the beginning of period t, the agent in charge during period t (Agent-t) inherits $w_t \geq 0$ units in the numeraire good from his "parent", i.e., Agent-(t−1). Agent-t allocates the inherited wealth to maximize the-end-of-the-period wealth. In addition, he earns some income, y, during period t. Then, at the end of period t, Agent-t consumes by c_t and bequests the rest, w_{t+1}, to his "child", i.e., Agent-(t+1).

There are two ways in which Agent-t could allocate the inherited wealth. First, he may run a non-divisible investment project, which converts F units of the numeraire good at the beginning of period into R units at the end of period. (One may interpret that F is the cost of education, and R is the increased lifetime earning from education.) Second, Agent-t may lend $x_t \leq w_t$ at the beginning of period t for rx_t at the end of period t.

By running the project, the agent earns $y + R$. If $w_t > F$, the additional income $r(w_t - F)$ can be earned by lending $w_t - F$. If $w_t < F$, the agent needs to borrow $F - w_t$ and hence must repay $r(F - w_t)$. Either way, the end-of-the-period wealth is equal to $y + R - r(F - w_t) = y + rw_t + (R - rF)$ by running the project. This is greater than or equal to $y + rw_t$ (the amount achieved by lending the entire wealth) iff

$$(1) \qquad R \geq rF. \qquad \text{(PC)}$$

Thus, the project return needs to be higher than (or equal to) the opportunity cost of running the project in order to make the agent *eager* (or *willing*) to invest into the project. This constraint (as well as the analogous constraints in the models discussed later) shall be called the profitability constraint (PC).

Even if (PC) holds with strict inequality and hence the agent is eager to invest, credit market imperfections might prevent the agent from investing. Here, we are not concerned about the question of why the credit market may be imperfect, but about possible consequences of the imperfections. So, we will introduce the imperfections simply by assuming that, for a variety of reasons, no more than a fraction, λ, of the project revenue, R, can be pledged to the lenders for the repayment. Then, the agent can generate the rate of return required by the lenders iff $\lambda R \geq r(F-w_t)$, which can be further rewritten as:

(2) $w_t \geq w_c \equiv F - \lambda R/r$. (BC)

Only when this constraint is met, the agent is capable of borrowing and investing. This constraint, as well as the analogous constraints in the models discussed later, shall be called the borrowing constraint (BC). One may also call it the wealth constraint because $w_c \equiv F - \lambda R/r$ represents the minimum level of wealth necessary for the agent to obtain external finance.[2] Another way of looking at this constraint is $b_t = F-w_t \leq \lambda R/r$. That is, borrowing is limited by the present discounted value of the pledgeable revenue of the project, $\lambda R/r$. Note that, if $\lambda = 0$, the agent would never be able to borrow and hence must self-finance the fixed cost entirely. On the other hand, if $\lambda = 1$, (BC) is never binding whenever (PC) holds. By setting λ between zero and one, this specification allows us to examine the whole range of intermediate cases between the two extremes.

[2]Some authors call a constraint analogous to eq.(2) “the collateral constraint,” while other authors call it “the cash flow constraint,” or “the liquidity constraint.” In doing so, they assume implicitly (and often without justification) that the borrower’s wealth held only in collateralizeable assets or in liquid assets could be used to satisfy the constraint. I avoid the use of the terms “collateral” or “liquidity” because (BC) here depends on the borrower’s net worth, independently of the borrower’s portfolio or liquidity holdings.

For the investment to take place, both (PC) and (BC) must hold. For the rest of this section, let us assume R > rF, so that the agent invests whenever (BC) holds.

To close the model, the bequest rule must be specified. We keep it simple by assuming that Agent-t splits the end-of-period wealth between his own consumption, c_t, and his bequest to Agent-(t+1), w_{t+1}, to maximize $u_t = (c_t)^{1-\beta}(w_{t+1})^{\beta}$. This implies that the bequest is equal to a fraction, β of the end-of-the-period wealth, so that the wealth accumulation (WA) of this household follows

$$(3) \qquad w_{t+1} = \begin{cases} \beta(y + rw_t) & \text{if } w_t < w_c \equiv F - \lambda R/r, \\ \beta[y + rw_t + (R-rF)] & \text{if } w_t \geq w_c \equiv F - \lambda R/r, \end{cases} \qquad \text{(WA)}$$

which may be viewed as a mapping from w_t to w_{t+1}, as illustrated by Figure 1. It has a constant slope equal to βr, except the jump at w_c. It is assumed that $\beta r < 1$ to ensure the existence of a steady state. The jump occurs because the credit market imperfection prevents the household (or its agent currently in charge) from investing into the profitable project if its wealth is below the threshold, w_c. Figure 1(a) depicts the case of $W_L \equiv \beta y/(1-\beta r) < w_c \equiv F - \lambda R/r < W_H \equiv \beta(y+R-rF)/(1-\beta r)$, so that $w_\infty = W_L$ if $w_0 < w_c$, and $w_\infty = W_H$ if $w_0 > w_c$. In words, its long run wealth depends on its initial wealth. This case thus shows the possibility that the household with low wealth may be caught in vicious cycles of poverty. Figure 1(b) depicts the case of $w_c < W_L < W_H$. In this case, the household will eventually cross the threshold, which enables it to borrow and invest. Nevertheless, it suggests that it might take a long time to escape the poverty due to the credit market imperfection. Either way, Figure 1 illustrates how the credit market imperfection contributes to persistence of poverty.

In this model, what determines the dynamics of household wealth is the initial level of its own wealth, but not the initial distribution of household wealth in the economy. In fact, the model describes one household in isolation, and hence its wealth dynamics can be traced independently of any other households in the economy. We should thus interpret what has been shown above as an "individual poverty trap" to distinguish it from the type of poverty traps discussed later.

Nevertheless, Figure 1 can be used to trace the evolution of wealth distribution in the economy. Let $G_0(w)$ denote the fraction of the households whose wealth is below w at the beginning of period 0. In Figure 1(a), the household wealth converges to W_L for the fraction $X_0 \equiv G_0(w_c)$ of the households, and to W_H for the rest, so that the distribution is polarized in the long run. In Figure 1(b), the dispersion may be initially magnified but then will start shrinking and eventually disappear, generating something akin to the Kuznets inverted-U curve. Indeed, Galor & Zeira (1993) have argued that a model like this is useful for understanding how the credit market imperfections affect the evolution of household wealth distribution. See also the textbook treatment by Acemoglu (2008; Ch.21.6) or Basu (1997; Ch.3.4). This type of analysis demonstrates that there is no need to have a model of interacting households to understand the evolution of wealth inequality across households over the process of development. So, anyone who criticizes it by saying that this is not a model of interacting households is missing the point. And there is no doubt that this insight is robust to an introduction of certain interactions among households into the model, as Galor & Zeira (1993, section 6) themselves have demonstrated.

At the same time, one should not read too much from this type of analysis. For example, models of this kind are often interpreted as implying that wealth inequality will lead to a lower output and growth. Again, consider the case of Figure 1(a), where the long run aggregate wealth converges to $X_0W_L + (1-X_0)W_H$, which is increasing in $X_0 \equiv G_0(w_c)$. Thus, the economy reaches its maximum potential in the long run with $X_0 = 0$, which could be achieved by eliminating initial wealth inequality if the initial average wealth of the economy is larger than the threshold. As Acemoglu (2008) pointed out, however, this is not a general result, and this type of analysis cannot make any clear predictions about the relationship between inequality and growth. To see why, consider instead the case where the initial average wealth is below the threshold. Then, eliminating initial wealth inequality means $X_0 = 1$, thereby ensuring that every household will reach W_L, and the economy's aggregate wealth would reach its minimum! In this case, the economy can grow only by making some households rich enough, which can be achieved only through an uneven distribution of initial wealth. To this, some might respond by saying that models of this kind imply that an uneven distribution is growth-enhancing in developing countries where average wealth is lower, while an even distribution is growth-enhancing in developed countries where average wealth is higher. However, this is not a robust prediction, either. It is a mere artifact of the simplifying assumption that there is only one investment project, which implies that the households never face any borrowing constraint above w_c, because it runs out of the investment opportunities. For example, modify this model so that the agent could choose among many investment projects, indexed by $j = 1, 2, \ldots,$ which share the same fixed cost, F, but their revenues, R_j, are increasing in j and the pledgeable revenues, $\lambda_j R_j$, are decreasing is j. Then, the steady state wealth sustainable by investing in the j-th project and the threshold level of wealth for financing the j-th project are both increasing in j. In this more general setup, there is no general prediction about inequality and growth. Empirically, a more equal distribution of wealth might be growth-enhancing among developed countries (although there is no consensus on this issue), but I imagine that nobody would be willing to argue that a more equal distribution is growth-enhancing among developed countries *because* they are so rich that an average household would face no borrowing constraint for financing any profitable project.

In summary, this type of analysis captures the first-order effects of credit market imperfections on the dynamics of individual household wealth. The key insight is that poor households might be caught in vicious circles of poverty due to credit market imperfections. This alone suggests the possibility that the rich may get richer and the poor may get poorer or the possibility that wealth inequality might initially increase and later decline over the course of development. As it stands, however, it is not useful for addressing the question of how wealth distribution or inequality affects the process of development, because wealth distribution per se plays no role in the analysis. To see this more clearly, ask the following question. "How will it affect the wealth dynamics of families currently living in this economy if a large number of penniless families immigrate to this economy?" In this type of models, the answer is "nothing" by assumption. If we are interested in any question about how the wealth distribution in general, and wealth inequality in particular, might affect the wealth dynamics of each household, we need to model explicitly how the households with different wealth levels might interact with each other in the economy.

Before proceeding, let us take a moment to address some modeling issues, which might have crossed the mind of some readers. Those eager to find out how the above model can serve as a building block in models of wealth distribution dynamics might want to go to the next section immediately.

Remark 1: It is possible to give any number of agency problem stories to justify the assumption that the borrower can pledge only up to a fraction of the project revenue. The simplest story would be that the borrower defaults strategically, whenever the repayment obligation exceeds the default cost, which is proportional to the revenue because a fraction of it would be dissipated in the borrower's attempt to default. Alternatively, one could assume that each project is specific to the borrower and requires his services to earn R; without his services, it earns only λR. Then, the borrower, by threatening to withdraw his services, could renegotiate the repayment obligation down to λR; see Hart (1995) and Hart & Moore (1994). One could also use the moral hazard model of Tirole (2005, Ch.3). Nevertheless, the reader may want to view this formulation simply as a black box, a convenient way of modeling the credit market imperfections without worrying too much about the underlying causes.

The careful reader might have also noticed that I avoid the use of terms such as "debt capacity," "interest rate," and "loan market," and instead use "borrowing limit," "rate of return," and "credit market." This is because we are concerned about possible consequences of credit market imperfections that arise from the difficulty of external finance in general. The borrowing constraint is introduced by the assumption that the borrowers cannot pledge the project revenue fully no matter what form of external financing are used, not by imposing an ad-hoc restriction on the menus of financial claims that they can issue. Indeed, the model is too abstract to make a meaningful distinction between equity, debt, and other forms of financial claims, which should be viewed as an advantage of the model; see Tirole (2005, p.119), who also argues for the benefits of separating the general issues of credit market imperfections from the questions of financial structure.

Remark 2: The specification of the bequest rule here has been widely used in this literature, including Banerjee & Newman (1993), Galor & Zeira (1993), Aghion & Bolton (1997), Piketty (1997), Matsuyama (2000, 2006). What simplifies the analysis is the assumption of "warm-glow altruism;" that is, each agent's utility depends on the amount given to the next agent, but not on the utility of the next agent. This means that the agent is not forward-looking when making the bequest. Under the alternative specification, "pure altruism," which assumes that each agent's utility depends on the utility of the next agent, but not on the amount given to the next agent, the current agent would be forward-looking in that it internalizes the consequence of its bequest decision on all the future agents. This effectively makes the agent choose the optimal consumption path for the dynasty, as is well-known in the Ricardian equivalence literature. While it may be more familiar to macroeconomists, the pure altruism specification has some drawbacks for our purpose. First, the steady state distribution of wealth in such a framework could be indeterminate even if the credit market is perfect, as pointed out by Becker (1980) and others.[3] This makes it harder to see the effects of credit market imperfections on wealth distribution. Second, in such a framework, the welfare of each agent is equal to the welfare of the dynasty, which makes the steady state wealth a less interesting object to study.

Some readers might object to the warm-glow altruism for the following reason. Suppose that the household starts slightly below the threshold, w_c, in Figure 1(a). Under the bequest rule assumed above, it will stay below the threshold, and its wealth converges to W_L. One might argue that a forward-looking agent should realize that, by making (possibly small) sacrifice of reducing his own consumption, he could bequest enough wealth so that all future agents would be able to make the profitable investment, and its wealth would converges to W_H. Such an objection does not invalidate the analysis, however. True, if the current wealth is only slightly below w_c, the agent might choose to reduce his own consumption to leave more wealth to the next agent, and the next few agents might do the same, so that the household wealth would eventually cross the threshold; see Greenwood & Jovanovic (1990) for a model with this feature. But there is clearly a trade-off. If the current wealth is further below w_c, the cost of reducing consumptions might become too high and/or it would take too many periods to cross the

[3]Contrary to the claim commonly made in the literature, the perfect credit market does not ensure the convergence of wealth across households. Those making such an assertion might be confused with the well-known result that the aggregate capital stock converges to the unique steady state from any initial condition in the one-sector neoclassical growth model. However, this result says nothing about the long-run distribution of the ownership of the aggregate capital stock across the households living in the same economy.

threshold, so that the household would not find it worthwhile accumulating wealth. In other words, even if the household consists of forward-looking agents with pure altruism, there might be a cut-off point below which the household finds it optimal not to accumulate. Such a cut-off point is known as "the Skiba point" in the optimal control problems where the forward-looking planner faces the convex-concave accumulation technologies; see Skiba (1978), Dechert & Nishimura (1983), Majumdar & Mitra (1982), and more recently, Ciccone & Matsuyama (1999) and Wagener (2003). Indeed, Buera (2006) applies this type of analysis to the context in which the household effectively faces the convex-concave investment return due to the credit market imperfections. If we used such a forward-looking household as a building block in an equilibrium model of interacting households, however, we would have to confront certain technical issues, particularly indeterminacy of equilibrium similar to those addressed by Matsuyama (1991) and Ciccone & Matsuyama (1996, 1999; section 5). A dynasty with the warm-glow bequest motive offers a simpler and more tractable alternative as a building block in models of interacting dynasties.

3. Interacting Dynasties: Labor Market Channel

One way in which the households with different levels of wealth can interact is through the labor market. Imagine that the rich households who invest and run the business have to hire workers. But, who become workers? They come from the poor households who cannot borrow to invest. The earning of the poor working households is equal to the wage rate, while the earning of the rich households, the profit, depends negatively on the wage rate. The wage rate is determined in equilibrium to keep the balance between the labor supply by the poor and the labor demand by the rich. This section introduces a model of interacting households with this feature, which is adopted from Matsuyama (2006). Section 4 will look at a model of households interacting through the credit market.

The economy consists of a unit mass of inherently identical households, each of which is an infinite sequence of one-period lived agents linked by inheritance. The only possible source of heterogeneity across households is their inherited wealth. Let $G_t(w)$ denote the share of the households whose agents have inherited strictly less than w at the beginning of period t.

As before, each agent seeks to maximize the end-of-the-period wealth. To this end, he makes investment as well as occupational decisions. There are two options: becoming a worker or an employer. (See Remark 5 for the effect of adding a third option, self-employment.) By becoming a worker, each agent earns the competitive wage rate, v_t. In addition, he lends the inherited wealth and earns the rate of return r per unit, which is assumed to be exogenous. (One may think of a storage technology, or of a small open economy, which cannot affect the market rate of return in the world market.) Thus, by becoming a worker, an Agent-t who inherited w_t will have $v_t + rw_t$ at the end of period t. (We keep the notation simple by setting $y = 0$.)

Alternatively, each agent may set up a firm and become an employer. Setting up a firm requires investing F units of the numeraire good at the beginning of the period. This (non-divisible) investment gives the agent access to the technology that produces $\phi(n_t)$ units of the numeraire good, which becomes available at the end of the period, by employing n_t units of labor at the competitive wage rate, v_t. The production function satisfies $\phi(n) > 0$, $\phi'(n) > 0$, and $\phi''(n) < 0$ for all $n > 0$, as well as $\phi(\infty) = \infty$ and $\phi'(\infty) = 0$. From the first-order condition for the profit maximization, $\phi'(n_t) \equiv v_t$, we may express the equilibrium employment of each firm as a function of the wage rate, $n(v_t)$, which is decreasing and satisfies $n(0) = \infty$. The gross profit from running a firm may be also expressed as a function of v_t, $\pi_t = \pi(v_t) \equiv \phi(n(v_t)) - v_t n(v_t) > 0$, which satisfies $\pi'(v_t) = -n(v_t) < 0$, $\pi''(v_t) = -n'(v_t) > 0$, and $\pi(0) = \phi(\infty) = \infty$. To keep it simple, let us assume that each employer can set up and manage at most one firm and that being an employer prevents the agent from earning the wage as a worker. (See Remark 6 for the effects of dropping these assumptions.)

If $w_t < F$, the agent needs to borrow $b_t = F - w_t$ at the gross rate of return equal to r in order to become an employer. If $w_t > F$, the agent can become an employer and lend $w_t - F$ at r. Either way, an agent who inherited w_t will have $\pi(v_t) + r(w_t - F)$ at the end of period t by becoming an employer. This is greater than or equal to $v_t + rw_t$ (the amount achieved by becoming a worker) iff

(4) $\pi(v_t) - v_t \geq rF$, or equivalently, $v_t \leq V$ (PC)

where $V > 0$ is a unique solution to $\pi(V) - V = rF$, and may also be expressed as $V = V(rF)$, a decreasing function satisfying $V(\infty) = 0$. If $v_t < V$, all agents prefer being employers to being workers. If $v_t = V$, they are indifferent. If $v_t > V$, then the wage is too high for the investment to be profitable; agents are better off being workers instead of being employers. Hence, eq.(4) is the profitability constraint (PC) for becoming an employer.

However, some agents may be unable to become employers due to the borrowing constraint (BC). As before, the imperfections are introduced by the assumption that the borrower/employer can pledge only up to a fraction of the profit for repayment, $\lambda\pi(v_t)$. Knowing this, lenders will allow a would-be employer to borrow only up to its discounted value, $\lambda\pi(v_t)/r$. Thus, the agent can set up a firm and become an employer only if

(5) $w_t \geq C(v_t) \equiv \text{Max}\{0, F - \lambda\pi(v_t)/r\}$, (BC)

where $C(v_t)$ is the threshold level of household wealth needed for the agent to become an employer. Agents become employers iff both (PC) and (BC) are satisfied. If one of these constraints fails, they become workers.

The labor market equilibrium can now be described. If $v_t > V$, (PC) fails; it is not profitable to be an employer, so every agent would become a worker; there would be an excess supply of labor. Thus, $v_t \leq V$ holds in equilibrium. Agents who inherited less than $C(v_t)$ violate (BC); they have no choice but to become workers. Only those whose inherited wealth is more than or equal to $C(v_t)$ can. And they are eager to become employers and hire $n(v_t)$ each if $v_t < V$ and are willing to do so if $v_t = V$. Thus, the labor market equilibrium (LME) condition is given by

$$(6) \qquad \frac{G_t(C(v_t))}{1 - G_t(C(v_t))} \leq n(v_t)\,; \qquad v_t \leq V, \qquad \text{(LME)}$$

where the first inequality can be strict either when G_t jumps at $C(v_t)$ or when $v_t = V$. Figure 2(a) illustrates Eq. (6). The downward-sloping curve shows labor demand per firm. The other curve shows labor supply per firm. Note that the supply curve is drawn flat at $v_t = V$, to capture the fact that all agents are indifferent between being employers and being employees.[4] If $v_t < V$, all agents prefer to be employers. As long as $G_t(C(v_t)) > 0$, however, the labor supply is positive because (BC) prevents some agents from becoming employers. In this range, this curve is generally upward-sloping, because a higher wage rate means a lower profit. This lowers the borrowing limit (when $\lambda > 0$), so that the agents need to be wealthier to become an employer. This means that more agents are unable to set up a firm, and forced to become a worker. The supply curve can be flat at $v_t < V$, as indicated by the dashed line. This occurs when a positive fraction of the agents inherited $C(v_t)$, so that $G_t(\bullet)$ jumps at $C(v_t)$. If the labor demand curve intersects with an upward sloping part of the labor supply curve, as depicted here, all agents borrow up to the borrowing limit. If they intersect at a flat part of the labor supply curve with $v_t < V$, some agents must be credit-rationed, meaning that they cannot borrow up to the limit, even

[4]Figure 2(a) depicts the situation where $G_t(C(V)) < 1$. If $G_t(C(v_t)) = 1$ for some $v_t < V$, then the labor supply curve stays strictly below the line, $v_t = V$. If $G_t(C(v_t)) < 1$ for all $v_t < V$, and $G_t(C(V)) = 1$, then the labor supply curve is asymptotic to the line, $v_t = V$.

though they want to do so and they are equally qualified as others.[5] This introduces an element of chance in the dynamics of household wealth. To deal with this situation, we would need to specify a rationing rule, which is inevitably ad-hoc. Or alternatively, we could specify some lotteries. However, we will not do so here, as we will focus on the properties of the model that are independent of any rationing rule specified. Nevertheless, it is important to keep in mind that a credit rationing might take place out of steady state, when the wealth distribution has a mass point.

Again, we close the model with the same bequest rule; each agent leaves a fraction β of end-of-the period wealth to the next agent. Thus, each household's wealth accumulates as:

$$(7) \qquad w_{t+1} = \begin{cases} \beta(v_t + rw_t) & \text{if } w_t < C(v_t), \\ \\ \beta(\pi(v_t) - rF + rw_t) & \text{if } w_t \geq C(v_t). \end{cases} \qquad \text{(WA)}$$

Figure 2(b) illustrates Eq. (7), which shows how w_t is mapped to w_{t+1} for a given v_t. The solid line graphs the map for the case where $v_t < V$, or equivalently, $\pi(v_t) - rF > v_t$. The map has a constant slope equal to $\beta r < 1$, except the jump at $C(v_t)$. Although all agents want to be employers when $v_t < V$, agents from poor households, whose wealth falls short of $C(v_t)$, have no choice but to work for the agents from rich households. The arrows indicate how a higher v_t shifts the map. A higher wage rate benefits the poor worker at the expense of the rich employer. Hence, the poor will have more wealth and the rich will have less wealth in the next period (though the rich still has more than the poor.) When $\lambda > 0$, a higher wage rate also makes the

[5] While some authors use the term, "credit rationing," whenever borrowing limits exist, here it is used to describe the situation in which the aggregate supply of credit falls short of the aggregate demand, so that some borrowers cannot borrow up to their borrowing limit. In other words, there is no credit rationing if every borrower can borrow up to its limit. In such a situation, their borrowing is constrained by their wealth, which affects the borrowing limit, but not because they are credit-rationed. This use of terminology is also consistent with the following definition by Freixas and Rochet (1997, Ch.5), who attributed it to Baltensperger: "some borrower's demand for credit is turned down, ev/en if this borrower is willing to pay all the price and nonprice elements of the loan contract."

threshold higher, as the investment becomes less profitable, which reduces the borrowing limit, and as a result, would-be employers would have to be richer. This suggests that a high wage rate is good for very poor household, which cannot borrow to become an employer in any case. It is bad for those in the middle, which could finance their investment at a lower wage rate (with prospects of a higher profit), but not at a higher wage rate (without prospects of a higher profit). Finally, the dashed line, $w_{t+1} = \beta(V+rw_t)$, depicts the map when $v_t = V$. In this case, all households earn V, equalizing the net earning of workers or employers, and hence there is no jump at C(V).

This completes the description of the model with interacting dynasties through the labor market. Given a wealth distribution at the beginning of period t, G_t, we can use eq. (6) or Figure 2(a) to determine the equilibrium wage rate, v_t, and profit, $\pi(v_t)$, as well as the occupational choice of the agents. Then, from eq. (7) or Figure 2(b), we can calculate the wealth distribution at the beginning of period t+1, G_{t+1}. By repeating this process, we can examine the joint evolution of the wage rate, profit, wealth distribution, and division of households between employers and workers.

Remark 3: The above model may be given an alternative interpretation. The numeraire good is produced competitively by CRS technology, $Y = F(K, L)$, where K is the aggregate supply of capital and L the aggregate supply of labor. When an agent becomes a worker, he supplies one unit of labor and earns $F_L(K,L)$. When an agent invests F, he produces one unit of capital and earns $F_K(K,L)$. This interpretation would give the same model by setting $\phi(n) \equiv F(1,n)$, as long as F satisfies $F_L > 0$, $F_{KL} > 0$ (or equivalently, $F_{LL} < 0$), $F(1, \infty) = \infty$, and $F_L(1, \infty) = 0$. This interpretation helps to make it clear that the poor and the rich supply complementary inputs. That is, the poor workers make the rich's investment more profitable and the rich's investment creates jobs, which increase the wage rate. It also helps to see the connection between this model and the single dynasty model of section 2, because the latter may be obtained by setting F(K,L) = RK.

3.1. Case of a Fixed Threshold

First, we consider a special case, $\lambda = 0$. This shuts down the credit market completely, so that would-be employers must self-finance their investments entirely, $C(v_t) = F$. What matters here is, however, not that the agents are unable to borrow, but that it makes (BC) independent of the wage rate, and hence, of the wealth distribution in the economy. The wealth distribution still affects the household wealth dynamics but only through its effects on the earnings of employers and workers. With the fixed threshold, this case captures the essence of Banerjee & Newman (1993); see also a simplified version of the Banerjee-Newman model by Ghatak & Jiang (2002).

Because $C(v_t) = F$, eqs. (6) and (7) are simplified to:

$$\frac{X_t}{1-X_t} \leq n(v_t)\,; \qquad v_t \leq V\,, \tag{8 (LME)}$$

where $X_t \equiv G_t(F)$, and

$$w_{t+1} = \begin{cases} \beta(v_t + rw_t) & \text{if } w_t < F, \\ \beta(\pi(v_t) - rF + rw_t) & \text{if } w_t \geq F. \end{cases} \tag{9 (WA)}$$

Figure 3(a) illustrates Eq.(8). If $X_t > n(V)/[1+n(V)]$, $X_t/(1-X_t) > n(V)$, so that $v_t < V$. Hence, only those whose wealth is less than F become workers, so that the labor supply per firm is vertical at $X_t/(1-X_t)$, independent of the wage rate. If $X_t \leq n(V)/[1+n(V)]$, the flat segment of the labor supply curve intersects with the labor demand curve, so that $v_t = V$, which means that the net earnings of the employers and the workers are equalized, so that even those above the threshold do not mind becoming workers. Figure 3(b) illustrates Eq.(9), with the arrows indicating the effects of a higher v_t. Note that the threshold does not move with the wage rate.

The dynamical system, defined by (8) and (9), is simple because it depends on the wealth distribution only through its value at F, $X_t \equiv G_t(F)$. Once we know X_t, we can determine v_t by Figure 3(a). This fixes the map shown in Figure 3(b), from which we could figure out X_{t+1}. Thus, the dynamics may be traced by following a single variable, X_t. To do so, it is useful to define v_c by $\beta v_c/(1-\beta r) \equiv F$. This is the minimum level of the earning that leads a household to eventually accumulate enough wealth to become an employer. We need to consider two cases separately, depending on the value of v_c.

Let us first consider the case of $v_c < V$, which can also be rewritten as $(1-\beta r)/\beta r < V(rF)/rF$, meaning that rF is not too large. Figure 4(a) depicts the situation under $X_t > X_c \equiv n(v_c)/[1+n(v_c)]$, so that $v_t < v_c < V < \pi(v_t) - rF$. Clearly, all households below the threshold in period t will stay below the threshold in period t+1, and all households above the threshold in period t will stay above the threshold. This means that $X_{t+1} = X_t$ and hence $v_{t+1} = v_t$. Thus, this is a steady state, $X_t = X_{t+1} = \ldots = X_\infty > X_c$ and $v_t = v_{t+1} = \ldots = v_\infty < v_c < V$. In this steady state, the households below the threshold form the working class (or the proletariat) and their wealth all converge to $\beta v_\infty/(1-\beta r) \equiv P(v_\infty)$, and those above the threshold form the entrepreneurial class (or the bourgeoisie) and their wealth all converge to $\beta(\pi(v_\infty)-rF)/(1-\beta r) \equiv B(v_\infty)$. (Here, P and B stand for the proletariat and the bourgeoisie.)

Compare this with the situation depicted by Figure 4(b), with $n(V)/[1+n(V)] < X_t < X_c \equiv n(v_c)/[1+n(v_c)]$, so that $v_c < v_t < V < \pi(v_t) - rF$. Then, the earning of the poor working households is high enough so that some of them will sooner or later succeed in crossing the threshold. Thus, X_t will decrease over time and hence v_t will increase over time. Eventually, $X_t < n(V)/[1+n(V)]$, beyond which point, $v_t = V$ will hold forever. This means that the wealth of all households follows $w_{t+1} = \beta(V+rw_t)$ and converges to $w_\infty = \beta V/(1-\beta r) > \beta v_c/(1-\beta r) \equiv F$.

To restate the above analysis, there are multiple steady states. One of them is characterized by equality where every household is rich, both in earning, $v_\infty = V = \pi(v_\infty) - rF$, and in wealth, $w_\infty = P(V) = B(V) = \beta V/(1-\beta r) > F$. There is also a trivial steady state with equality where every household is poor, $w_\infty = 0$, so that $X_\infty = 1$, and there is no labor demand, and hence $v_\infty = 0$. All other steady states are characterized by earning inequality between workers and employers, $0 < v_\infty < V < \pi(v_\infty) - rF$ and by wealth inequality, $P(v_\infty) < P(v_c) \equiv F < B(v_\infty)$, and the population becomes split between the bourgeoisie and the proletariat. The economy reaches the equal steady state with $v_\infty = V$, only if the initial distribution satisfies $G_0(F) \equiv X_0 < X_c$. If there are enough rich employers, their labor demand drives up the wage rate high enough to help the poor workers to eventually get out of poverty, and the class distinction disappears in the long run. In other words, the rich's wealth will spread or "trickle down" to all households. However, if $G_0(F) \equiv X_0 > X_c$, there are too many poor workers, which depresses the wage rate.[6] This keeps the poor households below the threshold, and the economy remains in steady state, $v_\infty < v_c < V$ and the population remains polarized between the bourgeoisie and the proletariat. In this case, the proletariat is caught in a poverty trap. Unlike the poverty trap in the single dynasty model, however, these households are trapped not so much because they are poor, but because there are too many of them. In this sense, this should be interpreted as a "collective poverty trap." (A trivial steady state, $X_\infty = 1$, may, of course, be viewed as an extreme case of a collective poverty trap.)

Again, recall the thought experiment of penniless families immigrating to this economy. Clearly, this is good news for the rich households. It is bad news for the poor households, because it reduces their earning and their chance of getting out of poverty.

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[6] Greenwood & Jovanovic (1990) also studied the mechanism in which how the wealth accumulation by the rich helps the poor crossing the threshold by driving up their wage rate. In their model, inequality does not disappear in the long run, because households are assumed to be heterogeneous in the labor endowment. They are primarily interested in generating Kuznet's Inverted-U curve.

The above analysis shows that the dynamics of wealth distribution exhibits path-dependence. That is, the long run distribution depends on the initial distribution. This result is usually described as "persistence of inequality." However, this should be interpreted with great caution. It is frequently interpreted as saying that greater initial inequality leads to greater long run inequality. Such an interpretation is false. To see why, suppose that, initially, there are $X_0 > X_c$ poor households all of which have the wealth equal to w^P_0, where $F > w^P_0 > (1-X_c/X_0)F$. The first condition, $F > w^P_0$, implies long run inequality, because more than X_c households are below the threshold. The second condition means $X_0w^P_0 > (X_0-X_c)F$ so that, by concentrating initial wealth among these poor households, we could push more than X_0-X_c of them above the threshold, so that less than X_c of them remain below the threshold. Such redistribution ensures long run equality, and yet, any Lorenz-consistent inequality measure, including the Gini index and the coefficient of variation, considers it as an increase in inequality. This example thus suggests that making initial distribution more unequal sometimes helps to achieve long run equality.

Nor should one interpret this type of models as saying that greater initial inequality will lead to a lower aggregate income and growth in the long run, although such an interpretation is quite common. To see this, notice that we may rank steady states by the total earning, $TE \equiv X_\infty v_\infty + (1-X_\infty)(\pi(v_\infty) - rF)$, or equivalently, by the aggregate wealth, $AW \equiv X_\infty P(v_\infty) + (1-X_\infty)B(v_\infty) = [\beta/(1-\beta r)]TE$. Simple algebra shows (see Matsuyama 2006, Section 3.4) that TE and AW are both higher in a steady state with a higher v_∞ and reaches its maximum at $v_\infty = V$, in the equal steady state. But, as the example in the last paragraph shows, one could sometimes reach the equal steady state by creating greater initial inequality. Again, this type of models has no clear prediction regarding the relationship between inequality and growth.

Let us briefly discuss the case of $v_c > V$, or equivalently $(1-\beta r)/\beta r > V(rF)/rF$, which holds when rF is sufficiently large. In this case, the equilibrium wage rate can never be high enough for the poor working households to get out of poverty. How about the rich who start above the threshold? To see this, define v_{cc} by $v_c \equiv \pi(v_{cc}) - rF$, which satisfies $v_{cc} < v_c < V$. Figure 4(c) depicts the situation where $X_t < X_{cc} \equiv n(v_{cc})/[1+n(v_{cc})]$, so that $v_{cc} < v_t \leq V \leq \pi(v_t) - rF < \pi(v_{cc}) - rF \equiv v_c$. Then, the net earning of the rich employers is too low so that some of them will sooner or later fall below the threshold. This implies that X_t will increase and hence v_t will decrease over time, until $X_t \geq X_{cc}$, and $v_t \leq v_{cc}$ hold, and hence $\pi(v_t) - rF \geq \pi(v_{cc}) - rF \equiv v_c$, as depicted by Figure 4(d). If any households are left above the threshold at that point, i.e., $X_t < 1$, they will stay above the threshold. Either way, once this situation is reached (or the economy starts from this situation), it will repeat itself, and $X_t = X_{t+1} = \ldots = X_\infty \geq X_{cc}$ and $v_t = v_{t+1} = \ldots = v_\infty \leq v_{cc}$. Again, there are multiple steady states, with path-dependence in the dynamics of distribution. However, the only steady state with equality in this case is a trivial steady state, where every household is poor, $w_\infty = 0$, $X_\infty = 1$, and $v_\infty = 0$. All non-trivial steady states are characterized by inequality.

In summary, this type of models is much richer in its implications, by allowing the household earnings to depend not only on its own wealth but also on the wealth distribution. Unlike the single dynasty model, it suggests the possibility that the poor households who start below the threshold may escape from the poverty, when there are enough rich households whose investment generate demand for their labor (a trickle-down). In other words, if the poor are caught in vicious circles of poverty, that is because there are too many of them (a collective poverty trap). The dynamics also exhibits path-dependence; the long run distribution depends on the initial distribution in a nontrivial way. However, this should not be interpreted as "persistence of inequality," because long-run equality may be sometimes achieved by making initial distribution more unequal. Furthermore, there is no general result suggesting that initial equality leads to higher aggregate income and growth.

3.2. Case of a Variable Threshold

Note that the above model always has a trivial steady state, $X_\infty = 1$, where every household is equally poor. This is a direct consequence of one peculiar feature of the model, i.e., the borrowing limit is exogenously fixed and does not adjust even if the investment becomes highly profitable. With the fixed borrowing limit, the threshold level of wealth is also fixed. If all households start below, nobody is rich enough to invest, so that nothing will happen in the economy, and hence nobody ever becomes rich enough to invest. Although such a steady state, often called "the underdevelopment trap," is frequently emphasized as an important message in this literature, it is not robust to a small change in the assumption, i.e., if the borrower could pledge even a small fraction of the profit, $\lambda > 0$. To see why, suppose that every agent's wealth were below the threshold. Then, nobody would invest, which means nobody would hire workers, which means that the wage rate must go down and the profit must go up, which in turns means that borrowers can pledge more to lenders, which raises the borrowing limit. Hence, in equilibrium, the threshold becomes sufficiently low such that some agents will be able to borrow and become employers. As the threshold separating the employers from the workers adjusts endogenously to maintain the balance between labor supply and labor demand, the notion of the rich and the poor becomes relative. The agents do not have to own a lot of wealth to become employers. Instead, what matters is their relative position in the distribution of wealth within the economy. They just need to own a relatively large amount of wealth to become employers.

Under the variable threshold, the effects of penniless families immigrating to this economy could be different, because their arrival improves the relative position of the poor households within the economy. As the presence of penniless immigrants lowers the wage rate, the profit, hence the borrowing limit goes up. This lowers the threshold such that the poor households would now be able to borrow and invest.

So, let us turn to the analysis of the case where $\lambda > 0$, so that $C(v_t) = \text{Max}\{0, F - \lambda\pi(v_t)/r\}$. As the variable threshold, the analysis of the dynamics becomes more involved so that we will focus on the steady state analysis. As just explained, there is no steady state with $X_\infty = 1$ so that there remain two types of the steady states to be distinguished.

The first type is what I call "classless society," which is the steady state with earning and wealth equality, where the wage rate satisfies $v_\infty = V = \pi(V) - rF$. In this steady state, wealth of each household follows the map, $w_{t+1} = \beta(V+rw_t)$, so that $w_\infty = \beta V/(1-\beta r)$. For such a steady state to be sustainable, the steady state wealth must be high enough so that all households could satisfy (BC) at $v_\infty = V$. This condition may be written as:

(10) $\quad w_\infty = \beta V/(1-\beta r) = B(V) = P(V) \geq C(V).$

Conversely, as long as (10) is satisfied, there exists a steady state, in which all households maintain the same level of wealth and are rich enough to be able to become employers. Furthermore, all households, whether they are workers or employers, earn the same net income, so that they are indifferent between becoming workers and employers so that (LME), eq. (8), holds with $v_t = v_\infty = V$. (Note that $G_\infty(C(V)) = 0$, because it is the share of households whose wealth is less than but not equal to $C(V)$.) Thus, (10) is the sufficient and necessary condition for this steady state.

The second type is what I call "class society," which are steady states with earning and wealth inequality, where the wage rate satisfies $v_\infty < V < \pi(v_\infty) - rF$. In these steady states, the employers earn higher net earnings than the workers. Such a steady state is sustainable only with an unequal distribution of wealth such that some households are rich enough to be employers and the others are too poor so that they remain workers. In such a steady state, the wealth of the rich bourgeoisie converges to the fixed point of the map, $w_{t+1} = \beta(\pi(v_\infty) - rF + rw_t)$, which may be written as $B(v_\infty) \equiv \beta(\pi(v_\infty) - rF)/(1-\beta r) \geq C(v_\infty)$, where the inequality means that they are indeed rich enough to be able to finance the investment. Likewise, the wealth of the poor proletariat converges to the fixed point of the map, $w_{t+1} = \beta(v_\infty + rw_t)$, which may be written as $P(v_\infty) \equiv \beta v_\infty/(1-\beta r) < C(v_\infty)$, where the inequality means that they are indeed too poor to be able to finance the investment. These conditions may be summarized as:

(11) $B(v_\infty) \equiv \beta(\pi(v_\infty) - rF)/(1-\beta r) \geq C(v_\infty) > P(v_\infty) \equiv \beta v_\infty/(1-\beta r)$,

Conversely, as long as this condition holds, (LME) holds with $X_\infty = X(v_\infty) \equiv n(v_\infty)/(1+n(v_\infty)) \in (0, 1)$. Therefore, eq.(11) is the sufficient and necessary condition for the existence of a steady state with $v_\infty < V < \pi(v_\infty) - rF$.

In Figures 5(a) through 5(c), these steady state conditions, (10) and (11) are illustrated by the graphs of $B(v_\infty)$, $C(v_\infty)$, and $P(v_\infty)$, all as functions of v_∞. The straight line with a positive slope depicts the wealth of the proletariat, $P(v_\infty)$; the convex, downward-sloping curve depicts that of the bourgeoisie, $B(v_\infty)$. The wealth gap between the two, $B(v_\infty) - P(v_\infty)$, shrinks as v_∞ goes up, and would disappear at $v_\infty = V$, where $P(V) = B(V) = \beta V/(1-\beta r)$. The third curve, the concave, upward-sloping one, depicts, $F - \lambda\pi(v_\infty)/r$, which is equal to $C(v_\infty)$, when it is positive. Note that this curve always hits the horizontal axis, so that $P(v_\infty) > C(v_\infty) = 0$ for a sufficiently small v_∞, which eliminates the trivial steady state, where every household is poor. As shown, there are only three generic ways in which $C(v_\infty)$ can intersect $P(v_\infty)$ and $B(v_\infty)$.

In Figure 5(a), it intersects $P(v_\infty)$ at $v^- > 0$ and $B(v_\infty)$ at $v^+ < V$. In Figure 5(b), it intersects $P(v_\infty)$ twice, first at $v^- > 0$ and then at $v^+ < V$, and it stays below $B(v_\infty)$ for all $v_\infty < V$. In Figure 5(c), it intersects neither $P(v_\infty)$ nor $B(v_\infty)$; it stays below $P(v_\infty)$ for all $v_\infty < V$. In Figure

5(a), the steady state with perfect equality does not exist because $P(V) = B(V) < C(V)$, violating (10). In both Figures 5(b) and 5(c), on the other hand, (10) holds, and hence there exists the equal steady state, where all households maintain $w_\infty = \beta V/(1-\beta r) \geq C(V)$.

In Figure 5(a), $P(v_\infty) < C(v_\infty) \leq B(v_\infty)$ over $(v^-, v^+]$, where $0 < v^- < v^+ < V$. Thus, (11) holds for $v_\infty \in (v^-, v^+]$. Each $v_\infty \in (v^-, v^+]$ corresponds to a steady state wage rate, with the share of the households in the proletariat, $X_\infty = X(v_\infty) \in [X(v^+), X(v^-))$, being strictly between 0 and 1. All these steady states are characterized by an unequal (two-point) distribution of wealth. The case of Figure 5(b) is similar except that (11) holds for $v_\infty \in (v^-, v^+)$ and $X_\infty \in (X(v^+), X(v^-))$. In Figure 5(c), on the other hand, $C(v_\infty) < P(v_\infty) < B(v_\infty)$ for all $v_\infty < V$, which means that (11) is never satisfied. That is, the only steady state is characterized by earning and wealth equality.

Figure 5(d) shows the parameter configurations for these three generic cases. Region A, B, and C correspond to Figure 5(a), 5(b), and 5(c), respectively; see Matsuyama (2006; Proposition 2) for the proof. Furthermore, on the horizontal axis (i.e., $\lambda = 0$), the border of Region B corresponds to the case of $V > v_c$ and the border of Region A corresponds to the case of $V < v_c$ in the model with a fixed threshold discussed in section 3.1.

In Region A, there is no steady state in which all households remain equally wealthy. In this case, the model predicts "emergent class structure." Even if every household starts with the same amount of wealth, the economy experiences "symmetry-breaking," and will be polarized into the rich bourgeoisie and the poor proletariat. The non-existence of an equal steady state means that a one-shot redistribution of wealth would not be effective; wealth inequality and the class structure always reemerge. Intuitively, with a large rF and a relatively small (but positive) λ, the wage rate must become sufficiently low to make it possible for some households to borrow and become employers. In order to keep the wage rate low, however, some households must stay poor, so that they are unable to borrow and are forced to work. In each steady state, the rich maintain their wealth partly because the poor work for them at a low wage. And the rich's demand for labor is not strong enough to pull the poor out of poverty. Across these steady states, a lower v_∞ implies that $B(v_\infty)$ is larger (i.e., the rich are richer); that $P(v_\infty)$ is smaller (i.e., the poor are poorer), and that $X(v_\infty)$ is larger (i.e., a larger fraction of the households is poor). This means that these steady states can be ranked according to the Lorenz criterion. That is, all Lorenz consistent inequality measures, including the Gini index and the coefficient of variation, agree that there is greater inequality in a steady state with a lower wage rate. Again, this is intuitive. The presence of a large working class depresses the wage rate, which favors the rich at the expense of the poor, which increases the wealth gap. A larger demand for labor by each rich employer can be met only when a small fraction of households belongs to the bourgeoisie. Note that the size of firms also increases with inequality. In a steady state with a lower wage, a smaller fraction of households belongs to the bourgeoisie, and each of them employs a larger number of workers.

In Region C, with a combination of a small rF and a high λ, there is a unique steady state, where every household is equally wealthy, and wealthy enough to be employers. Thus, the model predicts "global convergence," and "dissipating class structure." Starting from any initial distribution, inequality disappears in the long run, through a trickle-down mechanism, in which job creation by the rich pulls the poor out of poverty, and they can eventually catch up with the rich. Thus, there is no need for redistributing initial wealth to achieve long run equality. In this steady state, some agents work for others, but they do not mind doing so because their earnings are equalized, and those who employ operate on a relatively small scale, hiring a relatively small number of workers. In other words, the economy becomes a nation of the middle class, or "petit bourgeois", consisting of small proprietors and well-paid employees. And there is no need for redistributing wealth initially to reach this steady state.

In Region B, with a combination of a small rF and a small λ, both long run scenarios are possible. There is path-dependence, and whether the economy develops into a class society or not depends on the initial wealth distribution. In this sense, this case is similar to the model with the fixed threshold for the case of $V > v_c$, except that it has no underdevelopment trap.

Remark 4: Some readers might find it unsettling to see a continuum of steady states in Region A (and B), each of which is characterized by a two-point distribution. This feature of the model is a mere artifact of the simplifying assumptions that all the households are homogenous, except for inherited wealth, and that there are no idiosyncratic shocks. For example, if some idiosyncratic shocks are added by making the investment return random, as in the Banerjee-Newman model, a steady state might be characterized by a unique ergodic distribution of wealth, so that the question becomes whether the limit distribution is unimodal or bimodal (or "Twin Peaks" as Quah 1996 might call it). Such an extension would be also necessary to address "social mobility," the issue emphasized by Aghion & Bolton (1992). Alternatively, introducing a continuum of complementary job categories (instead of two, workers and employers), each of which requires different levels of specific human capital investments, could eliminate this kind of indeterminacy, as shown by Mookherjee & Ray (2003, 2010).

Remark 5: Matsuyama (2006, section 4) extends the above model to give the agents another investment opportunity, self-employment. Its effects, of course, depend on its profitability as well as the threshold level of wealth required for becoming self-employed. If the threshold is small, self-employment could provide the poor with an alternative to working for the rich (just like the subsistence wage option in the Banerjee & Newman model). When its threshold is relatively large but if it is profitable enough, self-employment could provide the rich with an alternative to the job-creating project, which could benefit the poor (just like the self-employment option in the Banerjee & Newman model). It turns out that, for most cases, this does not change the properties of the model fundamentally. But, there are two notable exceptions. In one case, which could occur in Region A or B, introducing self-employment does not affect the existing steady states, but creates a new one, where everybody becomes self-employed and their wealth stays at the level strictly lower than $P(v_\infty)$ or $B(v_\infty)$ in another steady state. Thus, this new steady state may be viewed as a poverty trap that could prevent the economy from reaching to another steady state where every household is richer. In the other case, which could occur in Region B, introducing the self-employment technology eliminates all unequal steady states, but there is no steady state with active self-employment, leaving the equal steady state with $v_\infty = V$ as the unique steady state. In this case, self-employment plays only a transitory role; the poor agents use it as a "stepping stone" to accumulate enough wealth to become as wealthy as employers.

Remark 6: In the above model, it is assumed that each agent can set up and manage at most one firm, and the amount of investment is fixed at F. In other words, the employment technology is not only subject to the minimum investment requirement, but also subject to diminishing returns. One might think intuitively that, without diminishing returns, the rich would invest more and operate many firms, until their labor demand would drive up the wage rate so much that poor workers could catch up with the rich, eliminating Region A. As shown in Matsuyama (2006, section 5), however, the non-existence of an equal steady state survives when we change the assumption so that the employment technology satisfies the constant returns after the minimum requirement. This is partly because a higher wage raises the threshold for becoming an employer, and partly because even the richest agents would expand its scale of operation up to the point where their borrowing constraint becomes binding, which means that each employer hires more workers, so that more agents have to be below the threshold.

4. Interacting Dynasties with a Variable Threshold: Credit Market Channel

In section 3.2, we looked at the model with a variable threshold, where households interact through the labor market. This section shows that similar results can be obtained in a model of households interacting through the credit market. Go back to the single dynasty model of section 2 and extend it by assuming that there is a unit mass of dynasties with different levels of wealth, which interact in the credit market where the equilibrium rate of return, r_t, adjusts endogenously in each period to keep the balance between the aggregate demand and aggregate supply of credit. In addition to the non-divisible investment, which converts F units of the numeraire good to R units, assume that they have access to (divisible) storage technology, which generates a small rate of return, $\rho > 0$. Due to the space limitations, I will merely sketch how this model works by highlighting its key features.

First, with the rate of return being variable, the threshold becomes variable, because (BC) may be written as $w_t \geq C(r_t) \equiv \mathrm{Max}\,\{0, F - \lambda R/r_t\}$, where $C(r_t)$ is increasing in r_t when positive. Second, the rate of return must satisfy $\rho \leq r_t \leq R/F$, because $r_t > R/F$ would imply that (PC) is violated so that no agent would want to borrow and invest, and if $r_t < \rho$ would imply that no agent would want to lend. Suppose $\rho < r_t < R/F$. Then, i) if $w_t < C(r_t)$, the agents become (reluctant) lenders because they cannot borrow; ii) if $C(r_t) \leq w_t < F$, they become borrowers, because they need to borrow and can borrow; and iii), if $w_t > F$, they become lenders, because they have more than enough to self-finance their own investment. The equilibrium rate of return is determined by the credit market equilibrium (CME), when

$$\text{(12)} \qquad \int_0^{\infty} w dG_t(w) = [1 - G_t(C(r_t))]F \qquad \text{for } \rho < r_t < R/F; \qquad \text{(CME)}$$

holds, where LHS is the aggregate supply of credit and RHS is the aggregate demand for credit. Otherwise, $r_t = \rho$ with excess supply of credit and $r_t = R/F$ with excess demand for credit. Given the equilibrium rate of return, each household's wealth accumulates as:

$$\text{(13)} \qquad w_{t+1} = \begin{cases} \beta(y + r_t w_t) & \text{if } w_t < C(r_t), \\ \beta(y + R - r_t F + r_t w_t) & \text{if } w_t \geq C(r_t). \end{cases} \qquad \text{(WA)}$$

Notice that the RHS of eq.(13) is decreasing in r_t when the household is a borrower, i.e., if $C(r_t) < w_t < F$, and increasing in r_t when the household is a lender, i.e., if $w_t < C(r_t)$ or $w_t > F$.

Again, starting from any initial distribution, G_0, we could use (CME) and (WA) repeatedly to trace the joint dynamics of G_t and r_t, and this allows us to characterize all the steady states for each parameter configuration.

First, consider the case with a sufficiently large R and/or β. Then, one could show that, once the households become rich enough to be able to invest, they accumulate enough wealth and they will eventually become lenders (i.e., their wealth will exceed F.) This will lower r_t, which reduces $C(r_t)$, which allows poor households to start borrowing and investing. And once they start investing, they will accumulate enough wealth, so they will no longer need to borrow and so on. As this process continues, the prosperity trickles down from the rich to the poor, until every household will eventually succeed in escaping the poverty trap. With global convergence to long run equality, this case captures the essence of the trickle-down mechanism shown by

Aghion & Bolton (1997). Note that this scenario requires that the rich eventually stop borrowing, as they run out of the investment opportunities, with $r_\infty = \rho$.

Second, consider the case where R and β are not so high so that, even if some households become rich enough to be able to borrow and invest, they never accumulate enough wealth to become lenders (i.e., their wealth never reach F.) In this case, they will never stop borrowing.[7] Thus, the rate of return is determined by the balance between the credit demand by the rich and the credit supply by the poor. Within this case, there are three possible sub-cases. In the first, there is global convergence to long run equality. That is, starting from any initial distribution, wealth inequality disappears in the long run with $r_\infty = R/F$. This is because the strong credit demand by the rich drives up the rate of return, which helps the poor lenders accumulate their wealth and eventually succeed in catching up with the rich. In this case, the wealth trickles down from the rich to the poor, as in Aghion & Bolton (1997), but the mechanism is quite different, and it is more similar to Region C of Figure 5(d). In the second, there is a symmetry-breaking to create long run inequality. That is, starting from any initial distribution (i.e., even if it is perfectly equal), the population will be split into the rich borrower and the poor lenders in the long run. In this case, the rich remain rich, in part because they have access to the cheap supply of credit offered by the poor, who is excluded from the profitable investment and have no choice to lend their tiny wealth to the rich. This case also suggests that a one-shot redistribution of wealth would be ineffective in eliminating inequality. This case thus resembles Region A of Figure 5(d). Finally, there is the third case, similar to Region B of Figure 5(d), where the long run distribution of wealth depends on the initial distribution of wealth, so that a one-shot redistribution of wealth can be effective in eliminating inequality.

[7]Alternatively, we can modify the model so that the rich will always find more profitable investment opportunities as they become richer, so that they never stop borrowing, as in the model of Matsuyama (2000).

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Figure 1: Wealth Dynamics in a Single Dynasty Model: Individual Poverty Traps

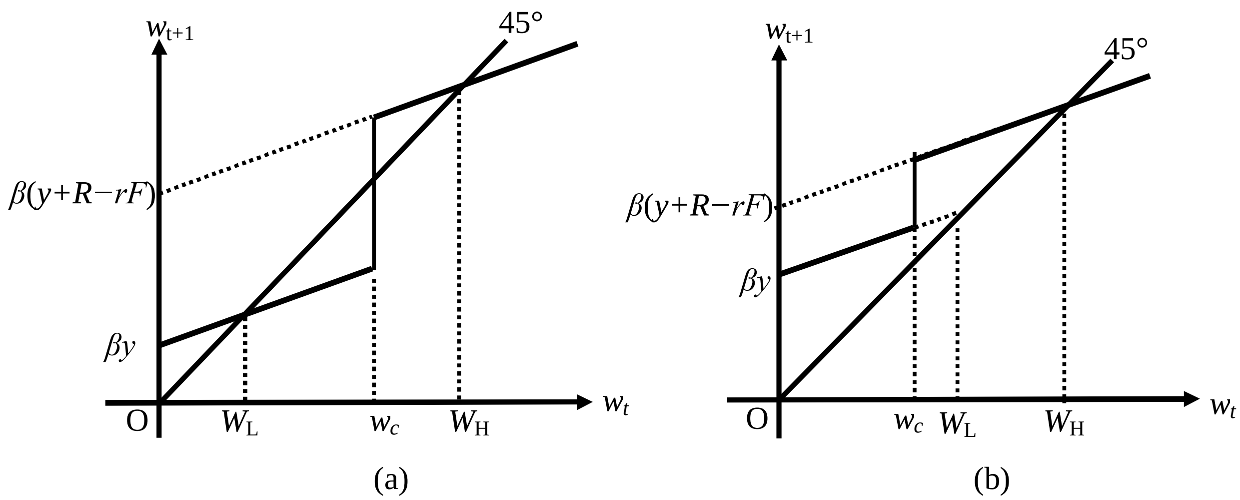
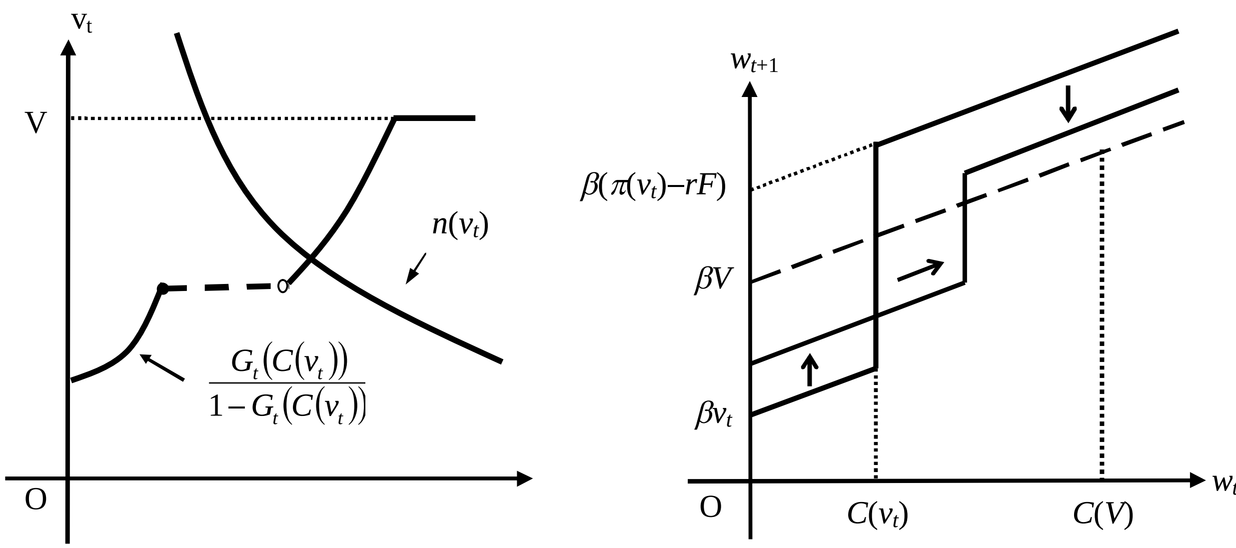


Figure 2: Labor Market Equilibrium and Wealth Dynamics in a Model of Interacting Dynasties



(a) (b)

Figure 3: Labor Market Equilibrium and Wealth Dynamics in a Model of Interacting Dynasties with a Fixed Threshold

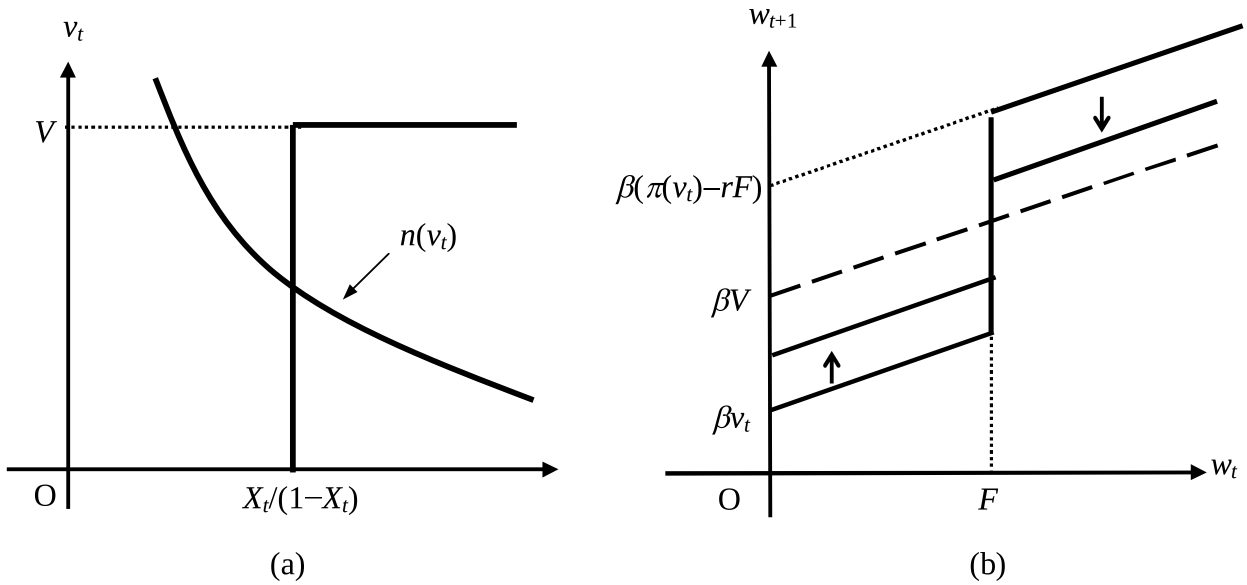


Figure 4: Dynamics in a Model of Interacting Dynasties with a Fixed Threshold

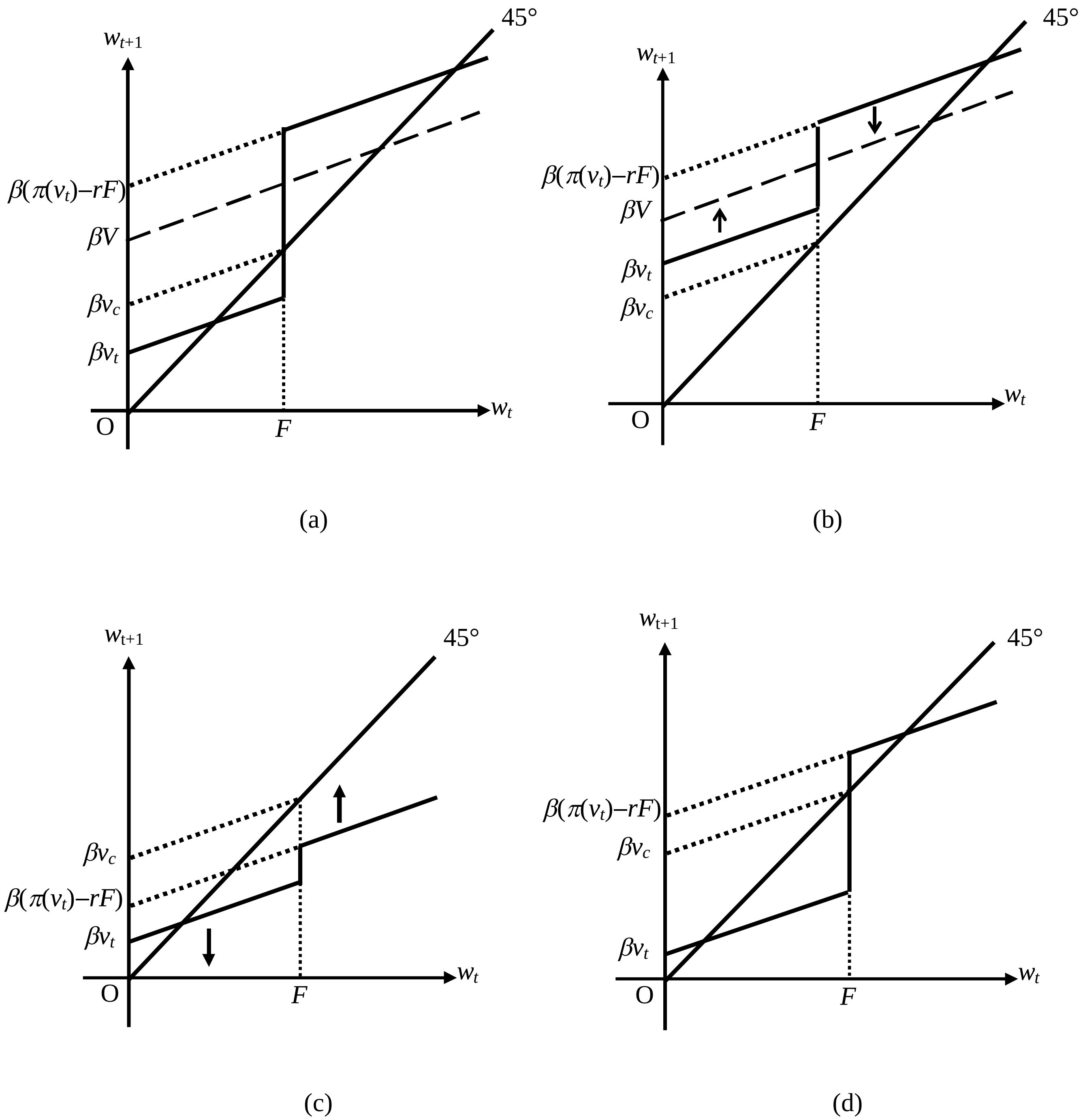


Figure 5: Three Generic Cases in a Model of Interacting Dynasties with a Variable Threshold